AQA

AS
Mathematics

MFP1 – Further Pure 1

Mark scheme

6360

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

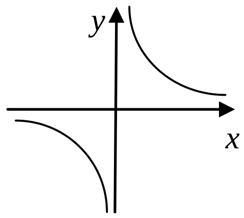
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)		**B1**	**1**	Rectangular hyperbola in 1st and 3rd quadrants with no part of the curve in the 2nd and 4th quadrants and no incorrect asymptotes shown. End points should not show any **significant** deviation away from the coordinate axes.
(b)(i)	$(x-3)(y+2)=16$	**B1**	**1**	ACF Apply ISW after a correct eqn if error in further rearrangement
(b)(ii)	$x=3,\quad y=-2$	**B1**	**1**	Both required. ACF
(c)	$(y-3)(x+2)=16$	**B1F**	**1**	Interchanging *y* and *x* in c's eqn of C_2 . Only ft if B0 in (b)(i). Apply ISW if error in further rearrangement.
	Total		**4**	

Q2	Solution	Mark	Total	Comment
(a)	$\mathrm{f}(x) = x - x^2 + \frac{2}{x} + \frac{3}{2}$			
	$\mathrm{f}(2) = 0.5$ (>0); $\mathrm{f}(2.5) = -1.45 = -\frac{29}{20}$ (< 0)	**M1**		Both values correct; condone -1.4 or $-1.4...$
	Since sign change (and f continuous in given interval), $2 < \alpha < 2.5$	**A1**	**2**	All values and working correct plus relevant concluding statement involving 2 and 2.5
(b)	$\mathrm{f}'(x) = 1 - 2x - \frac{2}{x^2}$	**M1**		Correct differentiation of at least 3 of the 4 terms of f(x).PI by $\mathrm{f}'(2) = -3.5$ seen/used
	$x_2 = 2 - \frac{\mathrm{f}(2)}{\mathrm{f}'(2)}$	**M1**		$2 - \frac{\mathrm{f}(2)}{\mathrm{f}'(2)}$ seen or explicitly used to indicate NR applied.
	$= 2 - \frac{2 - 2^2 + \frac{2}{2} + \frac{3}{2}}{1 - 4 - \frac{2}{4}}$	**A1F**		OE eg $2 - \frac{0.5}{-3.5}$. If incorrect ft only on c's f(2) value in part (a)
	= 2.143 (to 3dp)	**A1**	**4**	CAO Must be 2.143 NMS scores 0/4
	Total		**6**	
(a) **(a)** **(b)**	Condone 'root', 'solution', 'x' in place of α but **not** 'it'. Consult TL if values between 2 and 2.5 used instead. Accept 'signs alternate', '-1.45<0<0.5' as examples of alternatives to 'signs change' $2 \le \alpha \le 2.5$ OE would score A0 If $2 - \frac{\mathrm{f}(2)}{\mathrm{f}'(2)}$ is not written and $\mathrm{f}'(x)$ is incorrect, we must see substitution of 2 in $\mathrm{f}'(x)$ for the M1			

Q3	Solution	Mark	Total	Comment
(a)	…… = $2(-3+h)\{27-(-3+h)^2\}$ $y_Q = -2h^3+18h^2-108$	**B1**		OE with factor 2 or with factor -2, seen or used at any stage in **(a)**.
	Gradient = $\dfrac{2(-3+h)(27-(-3+h)^2)-(-108)}{-3+h-(-3)}$	**M1**		Use of gradient = $\dfrac{y_2-y_1}{x_2-x_1}$ OE to obtain an expression in terms of h with the expression in the denominator which would simplify to h or $-h$
	$= \dfrac{-2h^3+18h^2}{h} = -2h^2+18h$	**A1**	**3**	$-2h^2+18h$ OE in a factorised form obtained convincingly.
(b) (i)	As $h \to 0$, $-2h^2+18h \to 0$	**dM1**		OE eg $\lim_{h\to 0}$ [c's(ah^2+bh+c)] NB 'h = 0' instead of '$h \to 0$' gets **dM0**
	Grad. = 0 at P so P is a SP	**A1**	**2**	Dep on previous 4 marks scored. Grad. = 0 at P so P is a SP; must be referring to the curve; Final answer left as 'grad $\to$ 0 so P is a SP ' is **A0**
(b) (ii)	$y = -108$	**B1**	**1**	OE eg $y+108=0$
	Total		**6**	

(b)(i)	OE wording for '$\to$' eg 'tends to', 'approaches', 'goes towards'. Do NOT accept '='
(b)(ii)	$y = -108$ OE for the eqn of the tangent either stated (or found using any valid method).

Q4	Solution	Mark	Total	Comment
(a)	**Square** of a **real** number is **positive** (or 0) **Sum** of **squares** of two **real** numbers is **positive** (or 0)	**E1** **E1**	 **2**	OE OE Marks are indep and can be awarded in any order.
(b)	$\alpha+\beta=-\frac{3}{2}$	**B1**	**1**	$-\frac{3}{2}$ OE
(c)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$			
	$-\frac{7}{4}=\left(-\frac{3}{2}\right)^2-2\left(\frac{k}{2}\right)$	**M1**		$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ used to form a linear equation in *k* only. [or showing $2\alpha\beta=4$ and $\alpha\beta=\frac{k}{2}$ OE]
	$\frac{9}{4}+\frac{7}{4}=k\Rightarrow k=4$	**A1cso**	**2**	Must be no squared term immediately before the printed answer.
Altn	$2\left(-\frac{7}{4}\right)+3\left(-\frac{3}{2}\right)+2k=0$	**(M1)**		**Altn** $2(\alpha^2+\beta^2)+3(\alpha+\beta)+2k=0$ used
	$2k=\frac{7}{2}+\frac{9}{2}\Rightarrow k=4$	**(A1cso)**	**(2)**	Must be no squared term immediately before the printed answer.
(d)	$P=4+\left(-\frac{3}{2}\right)+\frac{1}{2}=3$	**B1F**		If incorrect, ft on '4.5 + c's answer **(b)**'
	$S=\alpha^2+\beta^2+\frac{\alpha+\beta}{\alpha\beta}$	**M1**		A correct expression for sum of roots in terms of known values, seen or used
	$S=-\frac{7}{4}+\frac{-1.5}{2}=-\frac{5}{2}$	**A1**		A correct value for the sum of the roots
	$x^2-(-2.5)x+3(=0)$	**M1**		Using x^2-Sx+P OE, **with** ft substitution of c's *S* and *P* non-zero values
	$2x^2+5x+6=0$	**A1**	**5**	ACF of the quadratic **equation**, but must have integer coefficients
	Total		**10**	
Eg **(a)** Eg **(a)** Eg **(a)** **(c)** **(d)**	'sum of two squares is positive' **(E0)(E0)**. If you square 2 real roots their sum could not equal a negative number so at least one root must be imaginary **(E0)(E1)** If α and β are both real then α^2 and β^2 must both be positive, but $\alpha^2+\beta^2$ is negative meaning one of them must be imaginary **(E1)(E1)**. A1cso so eg cands. who have $\alpha+\beta=\frac{3}{2}$ in (b) will likely still get $k=4$ but will not score the A1cso Consult team leader if candidate is using a valid substitution method… using eg $y=x^2+\frac{x}{2}$			

Q5	Solution	Mark	Total	Comment
(a)	$z^2 = p^2 + 6p\,\mathrm{i} + 9\,\mathrm{i}^2$	**B1**		$p^2 + 6p\,\mathrm{i} + 9\,\mathrm{i}^2$ OE
	$= p^2 + 6p\,\mathrm{i} - 9$	**M1**		$\mathrm{i}^2 = -1$ used at any stage in part **(a)**
	$z^* = p - 3\,\mathrm{i}$	**B1**		$z^* = p - 3\,\mathrm{i}$ seen or used
	$w = z^2 - 8z^* - 18p^2\mathrm{i}$ $= p^2 + 6p\mathrm{i} - 9 - 8(p - 3\mathrm{i}) - 18p^2\mathrm{i}$ $= p^2 - 8p - 9 + \mathrm{i}\left(6p + 24 - 18p^2\right)$			
	Re w = $p^2 - 8p - 9$	**A1**		OE eg $(p-9)(p+1)$
	Im w = $-18p^2 + 6p + 24$	**A1**	**5**	OE eg $6\left(-3p^2 + 4 + p\right)$ **SC** if last two marks are **A0 A0** then award SC1 mark for a correct X+iY form eg $p^2 - 8p - 9 + \mathrm{i}\left(6p + 24 - 18p^2\right)$
(b)	$p^2 - 8p - 9 = 0$	**M1**		c's real part equated to 0, seen or used. If cand also equates their Im part to 0 then **M0**
	$p = -1,\ \ p = 9$	**A1**		Correct two values of p
	$\Rightarrow -18p^2 + 6p + 24 = 0,\ \ -1380$ So only one non-zero value of w which is $-1380\,\mathrm{i}$	**A1**	**3**	Showing that the correct two values for p give $-1380\,\mathrm{i}$ as the only non-zero value
	Total		**8**	
(a) **(b)**	Re: $p^2 - 8p - 9 = 0$; Im: $-18p^2 + 6p + 24 = 0$ award SC tick1 in place of any A marks Likely method errors, Im part=0, Im part = Re part, Re part + Im part =0, all lead to two values for p one of which is $p = -1$……all these cases of wrong method result in 0/3 for part **(b)**.			

Q6	Solution	Mark	Total	Comment
(a)	$\cos(x-38^\circ) = -\cos 80^\circ = \cos 100^\circ$	B1		$\cos(x-38) = \cos 100$ OE PI by next line in soln or $x-38=100$
	$x - 38^\circ = 360^\circ n \pm 100^\circ$	M1		Ft c's $\cos(x-38) = \cos\lambda$ ie $x-38 = 360n \pm \lambda$ OE
	($x =$) $360^\circ n \pm 100^\circ + 38^\circ$	A1F		Ft c's λ ie ($x =$) $360n \pm \lambda + 38$ OE
	$x = 360^\circ n + 138^\circ$, $360^\circ n - 62^\circ$	A1	4	OE simplified form
(b)(i)	($\cos^2\left(\frac{5\pi}{12}\right) =$) $\frac{4-2\sqrt{3}}{8}$	B1	1	OE Most likely $\frac{2-\sqrt{3}}{4}$ direct from calc
(ii)	$\cos^2\left(\frac{5\pi}{12}\right) = \left(\sin\frac{\pi}{6}\right)\left(1-\frac{\sqrt{3}}{2}\right)$	B1		$\sin\frac{\pi}{6} = \frac{1}{2}$ stated or used
	$\Rightarrow \sin a\pi = 1 = \sin\frac{\pi}{2}$, $\left(a = \frac{1}{2}\right)$	B1		OE ie Any correct positive rational value for *a* which satisfies $\sin a\pi = 1$
	$\Rightarrow \sin b\pi = -\frac{\sqrt{3}}{2} = \sin\frac{4\pi}{3}$, $\left(b = \frac{4}{3}\right)$	B1		OE ie Any correct positive rational value for *b* which satisfies $\sin b\pi = -\frac{\sqrt{3}}{2}$
	$\cos^2\frac{5\pi}{12} = \left(\sin\frac{\pi}{6}\right)\left(\sin\frac{\pi}{2} + \sin\frac{4\pi}{3}\right)$ (*)		3	
	Total		**8**	

(a) Condone missing degree symbols throughout

(a) **ALT1** Using $\cos(x-38^\circ) + \cos 80^\circ = 2\cos\left(\frac{x-38+80}{2}\right)\cos\left(\frac{x-38-80}{2}\right)$

$\cos\left(\frac{x-38+80}{2}\right) = 0$, $\cos\left(\frac{x-38-80}{2}\right) = 0$ **B1** OE Need both

$\left(\frac{x-38+80}{2}\right) = 360n \pm \alpha$, $\left(\frac{x-38-80}{2}\right) = 360n \pm \alpha$ **M1** Need both. ft $\alpha =$ c's $\cos^{-1} 0$ $\neq 0$

$x = 2(360n \pm \alpha - 21)$, $x = 2(360n \pm \alpha + 59)$ **A1F** Need both, ft $\alpha =$ c's non-zero value of $\cos^{-1} 0$

$x = 2(360n - 111)$, $x = 2(360n + 69)$, $x = 2(360n + 149)$, $x = 2(360n - 31)$ **A1** OE simplified form

(a) Unlikely alternative: Correct eqn of the form $\sin(\mathrm{f}(x)) = \sin k$ **B1**

Full set of GS of c's [$\sin(\mathrm{f}(x)) = \sin k$] in the form f(*x*) = … **M1**

Full set of GS in form *x* = … , condone unsimplified, but only ft on a wrong *k* **A1F**

Correct full set of GS for *x* in simplified form **A1**

(b)

NB Labels *a* and *b* could be interchanged if eg cand works with the equiv form $\left(\sin\frac{\pi}{6}\right)\left(-\frac{\sqrt{3}}{2}+1\right)$

NB Check candidate's final answer for any other obscure correct alternatives for

$(\sin(a\pi) + \sin(b\pi)) = \frac{2-\sqrt{3}}{2}$ with *a* and *b* both **positive** rational numbers. Inform TL of any such cases

Q7	Solution	Mark	Total	Comment
(a)	$\sum_{r=1}^{n} r(4r+1)(4r-1) = 16\sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} r$	**M1**		$\sum_{r=1}^{n}(\alpha r^3+\beta r)=\alpha\sum_{r=1}^{n} r^3+\beta\sum_{r=1}^{n} r$ seen or used
	$\sum_{r=1}^{n} r(4r+1)(4r-1)-12n\sum_{r=1}^{n} r^2 =$ $\frac{16n^2}{4}(n+1)^2-\frac{n}{2}(n+1)-12n\frac{n}{6}(n+1)(2n+1)$	**dM1**		Substitution of correct expressions for at least two of the three summations
	$= \frac{n}{2}(n+1)[8n(n+1)-1-4n(2n+1)]$	**A1**		Correct factorisation at least as far as correct product of two linear factors and a correct expression.
	$= \frac{n}{2}(n+1)[8n^2+8n-1-8n^2-4n]$			Must see intermediate step
	$= \frac{n}{2}(n+1)(4n-1)$	**A1cso**	**4**	AG
(b)	LHS of eqn = $\frac{n}{2}(n+1)(4n-1)-\sum_{r=1}^{n} 57$	**M1**		PI by the next line
	$= \frac{n}{2}(n+1)(4n-1)-57n$	**B1**		$\sum_{r=1}^{n} 1 = n$ stated or used
	$= \frac{n}{2}(4n^2+3n-115)$			
	$= \frac{n}{2}(4n+23)(n-5)$	**A1**		PI by values 0 , $-\frac{23}{4}$, 5
	$= 0 \Rightarrow n = 5$	**A1**		$n=5$ as the only value.
	Exactly one value since *n* has to be (integer) $>$ 0	**E1**	**5**	OE Valid justification(s) for eliminating two of three values of *n*.
	Total		**9**	
(a)	Multiplying out brackets after the dM1 line: since AG, candidates must show sufficient intermediate terms to justify why the expression in the dM1 line simplifies to $2n^3+\frac{3}{2}n^2-\frac{n}{2}$ otherwise no **A** marks can be scored for such an approach.			
(b)	Candidates who use ‘=0’: 1st A1 would likely be in the form $n(4n+23)(n-5)=0$; if candidates have divided the cubic equation earlier by *n* without stating $n \neq 0$ then they would get $(4n+23)(n-5)=0$ and would not score the 1st A1 but can score the next A1.			
(b)	‘Cannot sum from 1 to 0’ is a valid justification for eliminating *n*=0 .			

Q8	Solution	Mark	Total	Comment
(a)	$q\boldsymbol{B} = \begin{bmatrix} 2q & 3q \\ -2q & q \end{bmatrix}$	B1		$q\boldsymbol{B} = \begin{bmatrix} 2q & 3q \\ -2q & q \end{bmatrix}$ seen or used. PI by correct values for *p*, *q* and *n*
	$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	B1		$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ seen or used. PI by correct values for *p*, *q* and *n*
	$\begin{bmatrix} p+2q & 5-2p+3q \\ 25-\frac{3p}{2}-2q & 15+q \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & n \end{bmatrix}$			
	$p+2q=n$, $5-2p+3q=0$ $25-1.5p-2q=0$, $15+q=n$	M1		Forming/using at least three linear equations, of which at least two are correct
	$p=10$, $q=5$, $n=20$,	A2,1,0	5	If not **A2** award **A1** if two values correct
(b) (i)	$\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$	B1	1	
(b) (ii)	$y=\sqrt{3}x=(\tan\theta)x \Rightarrow \theta=\frac{\pi}{3}$			
	$\begin{bmatrix} \cos\frac{2\pi}{3} & \sin\frac{2\pi}{3} \\ \sin\frac{2\pi}{3} & -\cos\frac{2\pi}{3} \end{bmatrix}$	M1		Correct matrix in any form; if matrix incorrect, ft on c's value for θ only if method for finding θ is shown
	$\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$	A1	2	Values must be exact
(b) (iii)	$\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} * & * \\ * & * \end{bmatrix}$	M1		Multiplication of c's matrices in the **correct order** to produce a 2×2 matrix
	$= \begin{bmatrix} -\frac{3}{2} & \frac{\sqrt{3}}{2} \\ \frac{3\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$	A1F	2	Only ft on one wrong matrix in previous two parts. Values must be in exact form.
	Total		**10**	

Q9	Solution	Mark	Total	Comment
(a)	$(0.2, 0)$	B1	1	OE . Condone eg 'y=0, x=1/5' but just the x-value scores B0
(b)	$-x+c=\dfrac{5x-1}{x-1}$	M1		Correct elimination of y
	$-x^2+x+cx-c=5x-1$	A1		
	$x^2-cx+c+5x-x-1=0$			or $-x^2-4x+cx-c+1=0$ [terms on the non-zero side may be in any order]
	$x^2+(4-c)x+c-1=0$ **(*)**	A1cso	3	AG . Must be in the form given in the Q
(c)(i)	Tangents so roots of (*) are real and equal $B^2-4AC=(4-c)^2-4(c-1)$	M1		Either B^2-4AC in terms of c or $B^2=4AC$ in terms of c or $B^2\geq 4AC$ in terms of c
	$(4-c)^2-4(c-1)=0$	A1		A correct equation obtained correctly where c is the only unknown
	$c^2-12c+20=0$ $c=2,\ \ 10$	A1		$c^2-12c+20=0$ or $(c-6)^2=16$ or correct two values of c from a correct eqn
	$y=-x+2,\quad y=-x+10$	A1	4	Correct two eqns of tangents ACF. Dep on M1 but not on previous two A1s NMS scores 0/4; Using differentiation scores 0/4.
(c)(ii)	$c=2\Rightarrow x^2+2x+1=0$ $c=10\Rightarrow x^2-6x+9=0$	M1		Substitution of either 2 or 10 into (*) OE to reach a quadratic in x with equal roots. PI by correct coordinates of both A and B
	Pts of contact: $(-1,3)\quad(3,7)$	A1		Correct coordinates of both A and B. Allow non-coordinate form if paired
	Gradient of AB = $\dfrac{7-3}{3-(-1)}$	M1		Correct method to find gradient of AB
	Gradient of AB = 1; Gradient of the tangents is -1	A1		Correct values of the gradients of both the line AB and the tangents;
	$\Rightarrow$right angle between AB and each of the two tangents$\Rightarrow AB$ and parts of the two tangents can form 3 sides of a square.	A1		Be convinced
	Area of the square = AB^2 = 32	A1	6	Dep on previous 5 marks scored
	Total		**14**	

(a)	Ignore (0,1)
(b)	eg $-x^2+cx+x-5x-c+1=0$ followed by the printed answer is an insufficient intermediate stage so does not score the A1cso.
(c)(i)	If starts with $b^2-4ac\geq 0$, max mark M1A0A0A1 unless cand states, for a tangent $b^2-4ac=0$ OE
(c)(ii)	After correct tangents and grad of AB=1, accept eg 'grad of tangent(s)=$-$ grad of AB' as OE to $m_{\tan}=-1$